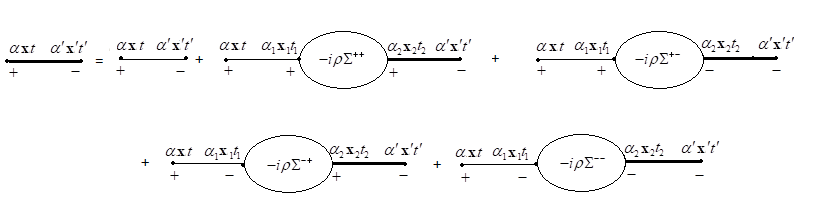
**Self-Energy**

**Elastic Solid Self Energy**

First I’ll look at the self energy in the context of a possibly time-dependent interaction. And then I’ll look at how it simplifies when interaction is constant.

**Self-Energy for GR**

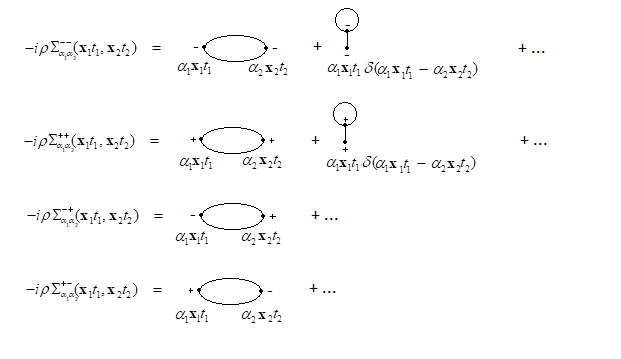
It seems plausible that the recursive equation for GR, and its concommitant self-energy, can be formulated as is done in the identical multi-particle case. And that everything would carry over straightforwardly. Presuming so, we can examine the self energy concept in the context of the general contour ordered GF, which may include time-dependent interactions. The self energy contains the energy corrections, lifetime corrections, and spectral weight of the excitations in question. It is the same as the information obtained from doing the TDPT in QM. So consider the self-energy expansion for G+-, for instance. I’ll use a classical, continuous, elastic solid for the sake of discussion. I’ll presume it to be homogeneous, but not isotropic necessarily, so that we’ll keep the index dependence of the Green’s functions, so that our exercise will apply more generally to photon GF’s, etc. The discussion will quite nicely parallel that for the multiple distinct particles in the QM folder.



(but note I defined the self energy bubble as -iρΣ, whereas it’d usually be defined as -iΣ, but I like how the equation turns out better) as usual we can simplify this equation down to:



Some of the self-energy terms, for the cubic interaction, would be:



(note the δ(α1**x**1t1 – α2**x**2t2) is three separate δ’s really – a Kronecker delta for the α’s and two Dirac deltas for the **x** and t respectively). And similarly for the other 3 GF. As in the QM file, we can write these four equations as a matrix equation:



with



Now we introduce a transformation to effectively diagonalize the equation:



Then the transformed matrix, G´, is:



The transformed self energies are, Σ´:



Turns out then, that the equations for GAR decouple, and we have simply (see QM folder):



Therefore, we *can* write out a self-consistent equation for GA, or GR in terms of just itself, though ΣA,R do involve GF0’s other than just G0A,R. Now we can operate on both sides with say (supposing for a second that our unperturbed system is isotropic, hence B is just a constant):



and then we’ll get:



which is an integro-differential equation from which one might attempt a non-perturbative calculation of G. You might think of the integral term as a sort of self consistent single particle potential approximation to the interaction. Going back to the highlighted equation, if we have a homogeneous system with time-independent interaction, so that G and Σ are dependent only on the difference of their temporal, spatial arguments (x1 – x2, t1 – t2), then we can take the spatio-temporal Fourier transform (∫d3x e-ikx ∫dt eiωt) of the equation (see Fourier transform file perhaps) and we’ll get:



Inversion of this equation yields (see QM Many Particle folder, but not hard),



where the 1/ and -1 operations are to be interpreted as matrix (in polarization space) inversion, in this context. If we can further presume isotropy, so that G and Σ are proportional to the unit tensor δαα´, then we can say,



And recalling for such a case, we found in the noninteracting file, that,



[I guess I’m calling the eigenfrequencies ωk, instead of Ωk – as I did in the QM distinct particles case – because in the continuum limit, the eigenfrequencies take an acoustic like linear spectrum] this comes to:



(presuming ΣR had an imaginary part, we could get rid of the iη) Given this form, we can obtain the spectral function:



The poles of the Green’s function determine the excitations, and so we will want to expand the Green’s function about its pole.



Suppose we find the root of this equation ω2 = [we’ll impicitly note that Σ is a function of ω2 because ω only appears in the G’s as squared]. Then we’ll expand the denominator of the Green’s function,



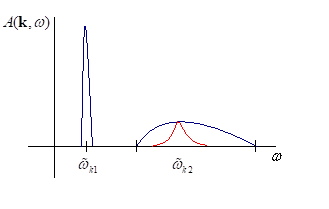
which we can write as:



Zk is called the renormalization factor. Z < 1 of course. The closer it is to 1, the closer the approximation is to a real eigenstate. Would the root **k** just be the frequency of oscillation about the new minimum? I guess not entirely because there could be an imaginary part as well. The spectral function will come to:



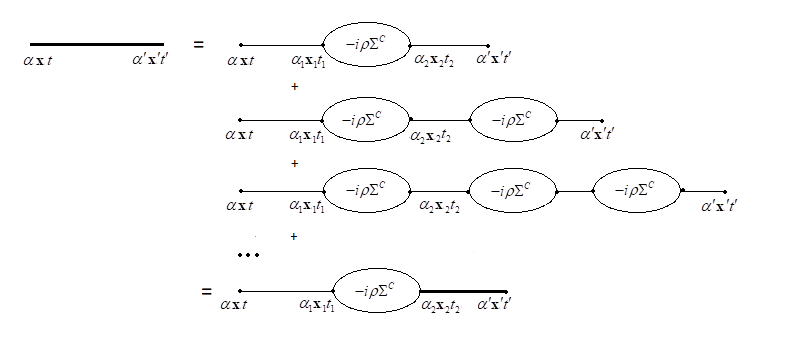
and again, the identifications make sense. Note how ImΣR must be less than 0 for A to be positive. A typical spectral function, A(**k**,ω), will look like below (this would be before we made the approximation about a single pole that we just did), in blue.



The presence of two peaks indicates that there are two excitations of the system – could be that our potential is of such a form that it has two minima, instead of the single one our unperturbed system has. The width of the peak is given by the ImΣ. So the delta function thing on the left would have that ImΣ = 0, while not so for the right hand curve. If we tried to approximate the latter with a Lorentzian, as we do above, we’d have the red curve, whose location gives the energy best associated with it, and whose width would give the scattering rate basically, and whose height would roughly give the renormalization factor, basically the amount that this would resemble a quasi-particle.

**Self-Energy for GC (time-independent interaction)**

The above applies still, but if things are time-dependent, then we have a self-consistent equation for GC as we saw in previous file. So I’ll formulate everything in terms of it. We can sum a subset of diagrams exactly. For instance,



which is equivalent to the equation:



Again let’s presume a homogeneous system so that G0, G, and Σ only depend on the difference **x** – **x**´, and t - t´. This would allow a spatio-temporal Fourier transform on our recursive equation. And we’d have:



We can solve for GC. We just recognize that the equation is equivalent to the matrix equation,



where G, G0 and Σ are matrices with components Gαα´, G0(αα´), and Σαα´. So then we can solve for GC,



and so,



This is usually written as:



where the 1/ and -1 operators are meant to be taken matrix inversion operators. If we can now further presume isotropy, then Gαα´, Gαα´(0), and Σαα´ should all be proportional to δαα´. And filling in our result for G0(αα´)C(k,ω), we get:



The poles of the Green’s function determine the excitations, and so we will want to expand the Green’s function about its pole, ω = k, defined by:



Suppose we find the root of this equation ω2 = [we’ll impicitly note that Σ is a function of ω2 because ω only appears in the G’s as squared]. Then we’ll expand the denominator of the Green’s function,



which we can write as:



Z is called the renormalization factor. Z < 1 of course. Would the root k just be the frequency of oscillation of the kth mode? I guess not entirely because there could be an imaginary part as well. Anyway, this allows us to identify the self-energy term as a correction term including the effects of the external potential. To the best degree possible it provides a description of the energy of GS as being comprised of harmonic oscillations with wave-vectors, **k**, and oscillation frequencies k.

**Photon Self Energy**

Just going to look at the causal GF self energy, presuming time-independent situation too.

**Self-Energy for GC (time-independent interaction)**

Now let’s look at the self-energy of the photon GF. Recall the free photon Hamiltonian, and let’s add some interaction to it.



In all cases of interest so far, it is Akλ, Akλ† which couples to other fields in the Hamilton. So the Green’s function was defined as [(H) means we’re looking at the fully time-developed operators]:



where (in fake Natural Gaussian units) we have:



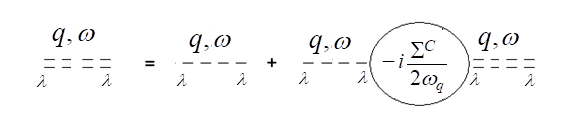
and this works out to:



Fourier transform yields,



And in a homogeneous isotropic medium, the different λ d.o.f. shouldn’t be coupled together. So we can write a self-energy expansion for D itself. It would look like this, in spatial/temporal Fourier space:



The recursion relation gives us:



which is:



which is, finally:



and recalling what the sum over the polarization vectors equals,



(basically the two polarization vectors are an orthonormal set perpendicular to the photon’s velocity – because EM waves are transverse of course) the G green’s function could be written as:

